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VISCOELASTIC EFFECTS OF DEFORMABLE ROLL COVER

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Abstract. *Roll coating is distinguished by the use of one or more gaps between rotating cylinders to meter a continuous liquid layer and to apply it to a flexible substrate. Of the two rolls that make a forward-roll coating gap, one is often covered by a layer of deformable elastomer. Liquid carried into the gap deforms the resilient roll cover. The complete understanding of the coupling between the liquid flow and roll cover deformation is vital to the optimization of this widely used coating method. Earlier works on deformable roll coating analyzed the action with both one-dimensional and plane-strain two-dimensional elastic models of the roll cover deformation. However, rubber and rubber-like materials used as roll covers do not behave purely elastically. Carvalho and Scriven (1996) analyzed the performance of deformable roll coating nips by means of lubrication approximation and a set of independent, radially oriented viscoelastic elements, the classic spring-dashpot combination from linear viscoelasticity theory. In order to test the accuracy of this simple approach and to evaluate the relationship between the empirical constants used in the model to the relevant physical parameters, a complete, two-dimensional, viscoelastic model has to be used. In this work, the flow between a rigid and a deformable rotating roll was examined by solving the complete Navier-Stokes system coupled with a plane-strain viscoelastic model of the roll cover deformation. The stress at each location of the roll cover was evaluated by an integral of the deformation along the material path of the point being analyzed. The model permits multiple relaxation time without extra computational effort. The equation system was solved by the Galerkin / finite element method. Results show how the viscoelastic properties of roll cover affect the performance of deformable roll nips.*

Key words: *roll coating, deformable cover, elastohydrodynamics, finite element method.*

1. INTRODUCTION

Roll coating process is characterized by liquid flow in a narrow gap or nip between rotating cylinders or rolls. The liquid is metered and then applied to a continous flexible substrate. Despite the variety of configurations, any such process can be broken into different parts, as described by Coyle et al. (1986). In order to understand the whole process, one needs to study the individual flows between pairs of rolls in forward and reverse mode.

The flow between two rigid rolls has been extensively studied in the past. However, usually one of the rolls of each gap is covered with a resilient layer that deforms during operation. The main purposes of using a deformable roll cover are to avoid the risk of clashing two hard rolls and to obtain much thinner films than ordinary can be achieved with rigid rolls. The deformation of the roll cover affects the shape of the boundaries of the coating flow. That flow generates pressure and viscous stresses, which deforms the roll cover. Hence, the viscous liquid flow and the deformation of the roll cover are coupled, which characterizes an *elastohydrodynamic* action.

Coyle (1988) analyzed this problem using Reynolds' equation for the liquid flow and a spring model to describe the roll cover deformation. In order to evaluate the limitations of the simple spring model and to relate the empirical spring constant to the roll cover properties, Carvalho and Scriven (1997) analyzed this situation by a complete two dimensional model. The liquid flow was described the Navier-Stokes equation and the deformation of the roll cover was described by a plane-strain, non-linear elastic constitutive equation.

Rubber covers used on deformable roll coating do not behave purely elastically. Their responses depend to a great extent on the stress history of the cover. Although this is well known, the roll cover materials are typically characterized only by an indentation test. Carvalho and Scriven (1996) extended the one-dimensional deformation model to include the viscoelastic behavior of rubber roll covers. The radially-oriented elements they used were simply a combination of springs and dashpots. To give an accurate description of the deformation of the compliant layer, a complete two-dimensional, finite deformation, viscoelastic formulation has to be used. This was done by Bapat and Batra (1984), who analyzed the deformation of a rubber cover roll indented by a rigid cylinder. They used the constitutive relation proposed by Christensen (1980). Their work was on dry contact between cylinders, i.e. there was no liquid flow, as in the case analyzer here.

In this work, the flow between a rigid and a deformable rotating rolls fully submerged in a liquid pool is studied. The flow is described by the complete Navier-Stokes equation and the roll cover deformation by a non-linear viscoelastic model, the same used by Bapat and Batra (1984) in dry contact. The goals were to study the effect of the viscoelastic behavior of the roll cover on the gap performance.

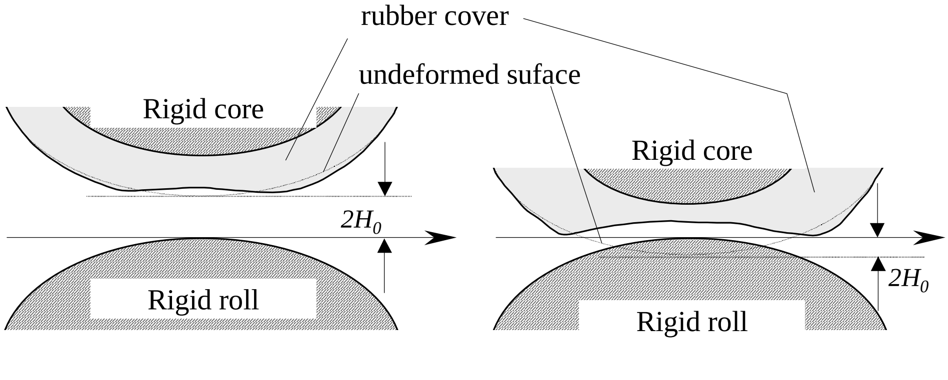

Figure 1: Sketch of flow between two rolls fully flooded in a liquid pool: (a) Detail of the nip region in positive gaps, i.e. clearance of $2H_0$ between undeformed rolls; (b) Detail of nip region in negative gaps, i.e. interference of $2H_0$ between undeformed rolls.

2. DEFINITION OF THE SYSTEM OF EQUATIONS

The liquid flow between a rigid and a deformable roll, both rotating and fully submerged in a pool is sketched in Fig.1. Roll A is rigid and roll B is covered with a viscoelastic rubber layer. The roll surfaces move in the same direction in the gap region. If the center-to-center distance is larger than the sum of the roll radii, there is a clearance between the undeformed roll surfaces; such situations are called *positive gaps*. If the center-to-center distance is smaller than the sum of the roll radii, the rolls would interfere were they undeformable; such situations are called *negative gaps*. For convenience, both the clearance and interference between undeformed rolls are called $2H_0$, as sketched in Fig.1.

2.1. Equations of Liquid Flow and Solid Deformation

The governing equations give rise to a free boundary problem, because the position of the deformable roll surface is unknown a priori. The basis of treating such problems is presented by Kistler and Scriven (1984), Sackinger et al. (1996), and Carvalho and Scriven (1997).

Figure 2 shows the domain of calculation. There is a liquid domain Ω_L, where the differential equations that describe the liquid motion are solved, and a solid domain Ω_S, where the differential equations that describe the solid deformation are solved.

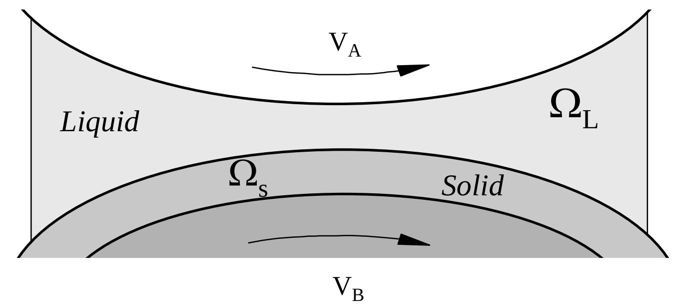

Figure 2: Sketch of domains for flow between a rigid and a deformable roll.

The motion of the liquid is described by the Navier-Stokes equation and continuity equation of incompressible flow

$$\rho \mathbf{v} \bullet \nabla \mathbf{v} - \nabla \bullet \boldsymbol{\sigma} = 0 \quad \text{and} \quad \nabla \bullet \mathbf{v} = 0 \tag{1}$$

together with appropriate boundary conditions. ρ is the density of the liquid and $\boldsymbol{\sigma}$ is represents the Cauchy stress tensor.

The rubber is taken to be incompressible and the acceleration and body forces of the resilient layer are neglected. With these assumptions, the differential equations that describe the roll cover deformation are

$$\nabla \bullet \boldsymbol{\sigma} = \mathbf{0} \text{ and } \det(\mathbf{F}) = \mathbf{1} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\mathbf{F}$ is the deformation gradient tensor. To complete the formulation, the stress has to be related to the deformation by an appropriate constitutive relation. Carvalho and Scriven (1997) used the Mooney-Rivlin equation:

$$\boldsymbol{\sigma} = -\pi \mathbf{I} + \beta_0 \mathbf{B} + \beta_1 \mathbf{B}^{-1}$$

where π is a scalar, pressure-like function that is related to the incompressibility constraint, just like the pressure in an incompressible liquid is related to the continuity equation. β_0 and β_1 are the elastic constants of the material.

In this work, the viscoelastic behavior of rubber materials is taken into account. The response of this type of material to a stress relaxation test is shown in Fig. 3. In particular, a non-linear viscoelastic equation proposed by Christensen (1980) is used. The stress is a function of the entire deformation history:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{F}\left\{\beta_0\mathbf{I} + \beta_1\left[\mathbf{E} - \frac{1}{\lambda}\int_{-\infty}^{t}\exp[-(t-\tau)/\lambda]\mathbf{E}(\tau)d\tau\right]\right\}\mathbf{F}^{\mathbf{T}} \quad (3)$$

$\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathbf{T}}$ is the left Cauchy-Green tensor, $\mathbf{E} = \frac{1}{2}\left[\mathbf{F}^{\mathbf{T}}\mathbf{F} - \mathbf{I}\right]$, and λ is the relaxation time constant of the material. The component inside the time integral represents the memory effect of the roll cover material. The intensity of the viscoelastic behavior can be measured by the amount of stress relaxation, that is proportional to the ratio $\beta_1/(\beta_1+\beta_0)$, and by the time that it takes the stress to completely relax, that it proportional to λ. The elastic response is recovered when the $\beta_1/(\beta_1+\beta_0)\to 0$ and $\lambda\to\infty$.

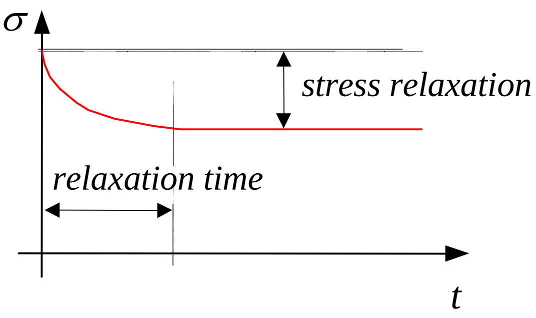

Figure 3: Response of a viscoelastic material to a stress relaxation test.

The boundary conditions used for the fluid flow and solid deformation are the usual ones: no-slip and no-penetration at solid walls, force balance at the resilient solid / liquid interface. The synthetic inlet and outlet boundaries are placed far enough from the region of interest that the liquid pressure is taken to be constant in both locations and the stress state of the roll cover is assumed to be completely relaxed upstream the roll nip. This hypothesis is valid if the relaxation time of the roll cover rubber is shorter than the time that it takes a material point to travel from the exit of the nip back to entrance. If this assumption is not valid, the problem becomes transiente and it is not analyzed here.

The time integral of the deformation history can be transformed to a space integral, as indicated below:

$$\int_{-\infty}^{t}\exp[-(t-\tau)/\lambda]\mathbf{E}(\tau)d\tau = \int_{0}^{l}\frac{\exp[-(l-s)/\lambda V(s)]}{V(s)}\mathbf{E}(s)dS$$

where $V(s)$ is the velocity of the point at each position s. The details of the kinematic of the roll cover is presented by Carvalho and Scriven (1997).

The dimensionless parameters that govern this situation are:

- Reynolds Number $\mathrm{Re} = \rho VR / \mu$,
- dimensionless undeformed clearance or interference H_0 / R,
- dimensionless roll cover thickness L / R,
- Elasticity Number $Es = \mu V / (\beta_0 + \beta_1) R$,
- elastic constant ratio $\beta_1 / (\beta_0 + \beta_1)$,
- Deborah Number $De = \lambda V / R$, and
- speed ratio $S = V_A / V_B$.

2.2. Solution Method

The position of the deformable roll suface is unknown *a priori*, it is part of the solution. In order to solve this free boundary problem using standard techniques, the set of differential equations posed in the unknown physical domains Ω_L and Ω_S have to be transformed to an equivalent set defined in a suitable known reference domain Ω_0, as illustrated in Fig. 4. The transformation used here to map the liquid physical domain Ω_L into the reference domain Ω_0, represented by $\mathfrak{F}_1$ in Fig. 4, relies on elliptic partial differential equations to relate points of both domains. Details of this procedure are given elsewhere (de Santos, 1990; Carvalho and Scriven, 1997). For the solid domain, it is convenient to map the equation system defined in the deformed configuration to the zero-stress configuration $\overline{\Omega}$, that serves as the reference configuration. This is accomplished by using Piola's transformation. From the numerical point of view, it is convenient to integrate the equations of both liquid and solid domain over the same reference domain Ω_0. For that, a mapping from Ω_0 to $\overline{\Omega}$ has to be constructed. It is represented by $\mathfrak{F}_2$ in Fig. 4. Unlike $\mathfrak{F}_1$, this mapping is known and it simply represents a change of domain of integration.

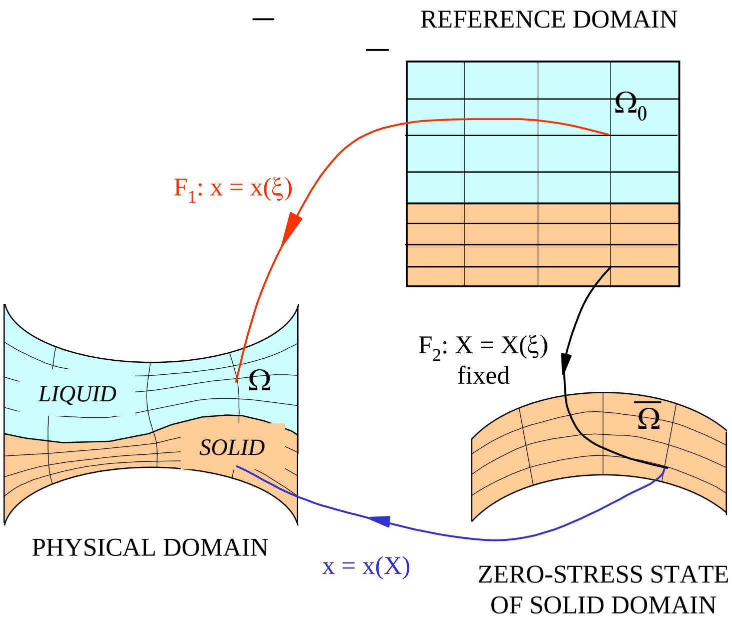

Figure 4: Mappings from the liquid domain to the reference domain (Ω_L - Ω_0); from the current solid configuration to the zero-stress state (Ω_S - $\overline{\Omega}$); and from the zero-stress state to the computational reference domain ($\overline{\Omega}$ – Ω_0).

The system of partial differential equations is solved by the Galerkin / Finite Element Method. The weighting functions associated with the momentum, mesh generation in the liquid domain, and deformation of solid domain equations were biquadratic; and the ones associated with the incompressibility constraint of both the liquid and solid were piecewise linear discontinuous functions.

The resulting non-linear set of equations was solved by Newton's method, which converges quadratically close to the solution. At each Newton iteration, the linear system of equation was solved by a multifrontal method, e.g. UMFPACK.

Unlike differential viscoelastic models, the integral model of eq.(3) to describe the viscous behavior of rubber like materials does not increase the number of degrees of freedom of the problem, when compared to the elastic description. However, the banded structure of the Jacobian matrix is lost, because the stress at a given point depends on the deformation along the entire pathline of the point. The domain of liquid flow was divided into 150 elements, and the domain of solid deformation, into 100 elements. The number of simultaneous equations was 5034.

3. RESULTS

The flow through the deformable gap was evaluated at different clearances and interferences H_0 / R, elasticity number $Es = \mu V / (\beta_0 + \beta_1) R$, Deborah number $De = \lambda V / R$, and elastic constant ratio $\beta_1 / (\beta_0 + \beta_1)$. The speed ratio S and the dimensionless roll cover thickness were kept constant, e.g. $S = 1$ and $L / R = 0.05$.

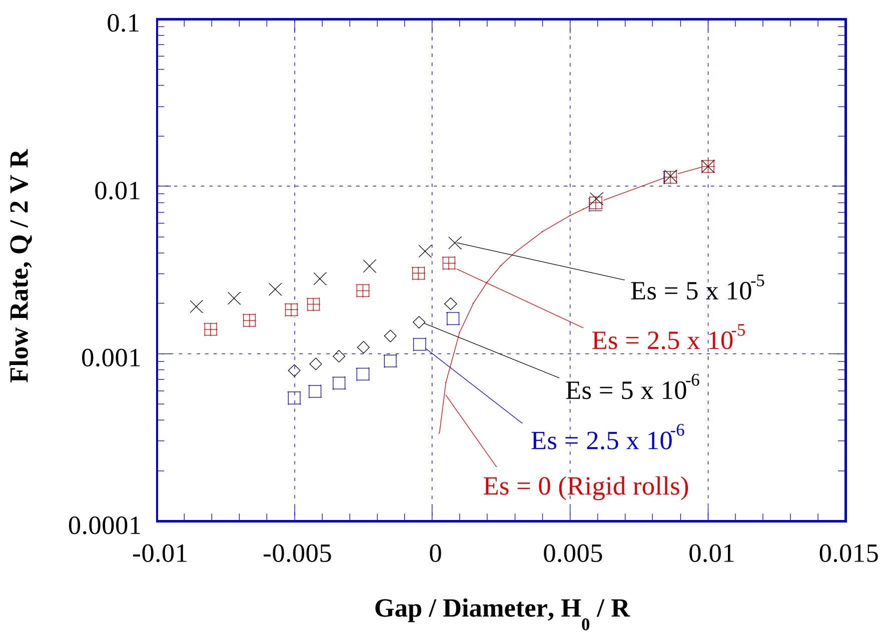

Figure 5: Dimensionless flow rate predictions at different gaps and elasticity numbers. The continuous line represents the solution obtained with rigid rolls.

Figure 5 shows the dimensionless flow rate $q^* \equiv Q / 2VR$ as a function of the center-to-center position, characterized by H_0 / R, at different elasticity numbers. Deborah number approached infinity, i.e. the rubber was considered an elastic material. At large enough positive gaps, the curves for different elasticity numbers merge into a single line that matches the rigid-roll predictions, because the liquid pressure is not high enough to deform the rubber cover. As the rolls are pushed against each other, the flow rate falls. At negative gaps, the flow rate sensitivity to roll position becomes relatively small. This is very important for roll coating operations. It means that the variation of the liquid layer thickness caused by roll run-out (out-of-roundness) is much smaller when a deformable roll is used and it diminishes as the rolls are pressed against each other. At a fixed center-to-center distance, the softer the roll, i.e. the larger the elasticity number, the larger the flow rate. Figure 6 shows the liquid and solid domains at different gaps (positive and negative). If the rolls are far apart, the pressure is not strong enough to deform the roll cover. As the rolls are pressed against each other, the roll cover deforms more and more. At large interference (negative gaps), the liquid layer between the two rolls is so thin as to be almost impreceptible in the plot.

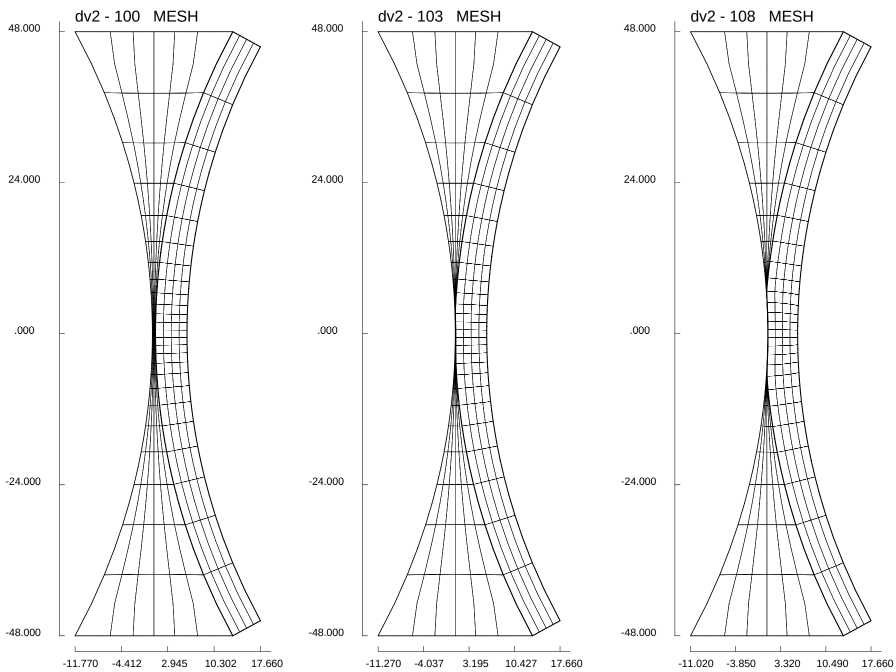

Figure 6: Sequence of deformable roll deformation at different positive and negative undeformed gaps.

The effect of the elastic constant ratio on the dimensionless flow rate is shown in Fig. 7. The predictions were obtained at $H_0 / R = -2\times10^{-3}$, $Es = 10^{-4}$, and $De = 10^{-2}$. As the stress relaxation becomes larger, i.e. as the elastic constant ratio rises, the amount of liquid that flows through the gap increases. The effect of the Deborah number is illustrated in Fig. 8. The predictions shown are at $H_0 / R = -2\times10^{-3}$, $Es = 10^{-4}$, $\beta_1 /(\beta_1 + \beta_0) = 0.1$ and 0.3. If the time response of the rubber material is very slow, i.e. at large Deborah, the prediction approaches that of an elastic material. As the reponse time of the rubber falls and Deborah number decreases, the flow rate between the rolls increases because the rubber cover has enough time to relax to a lower level of stresses.

The effect of viscoelasticity, as represented by Deborah number, on the pressure distribution is illustrated in Fig. 9. As the Deborah number falls, the rubber has more time to relax and consenquentely the pressure peak falls and shifts upstream. Scheuter and Pfeiffer

(1969) observed experimentally such shifting of the pressure distribution in dry rolling with rubber-covered rolls.

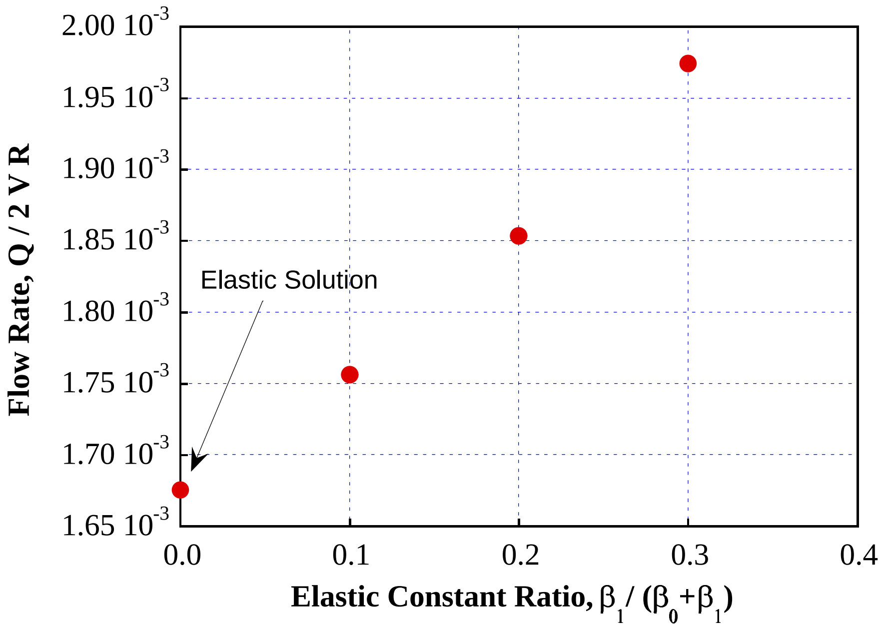

Figure 7: Effect of intensity of stress relaxation, characterized by the elastic constant ratio, on the flow rate through through the roll nip.

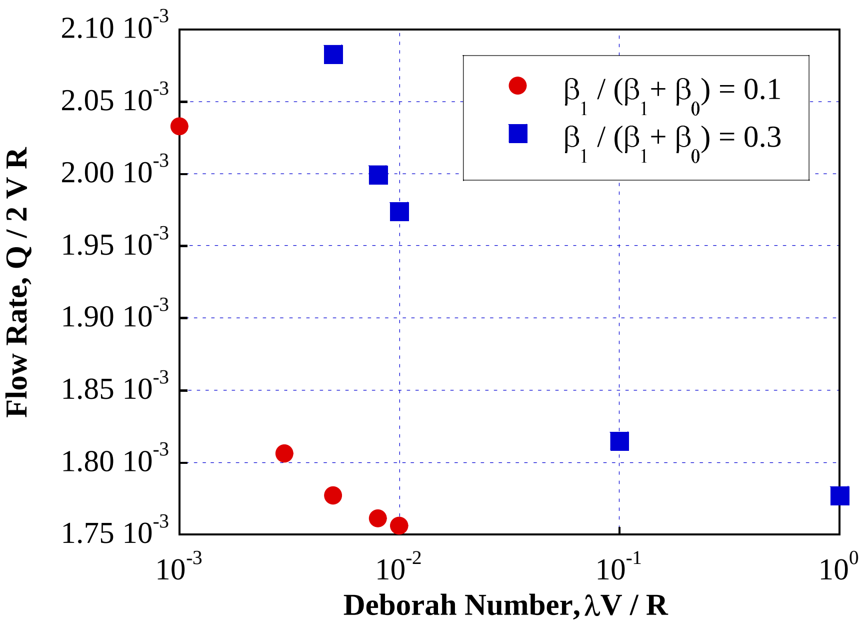

Figure 8: Prediction of flow rate versus Deborah number at different elastic constant ratio.

4. FINAL REMARKS

Flow between a rigid and a deformable roll was analyzed. The viscous liquid flow and the elastic deformation of the roll cover are coupled, which constitutes na elastohydrodynamic action.

A complete two-dimensional formulation of the situation was used for both the liquid flow and for the viscoelastic roll cover. The liquid flow between the rotating rolls was described by the Navier-Stokes system of equations. The deformation of the compliant roll cover was described by a plane-strain model of non-linear incompressible viscoelastic material. The formulation can be improved by considering a multi-relaxation time constant model, which give a better description of elastomeric materials.

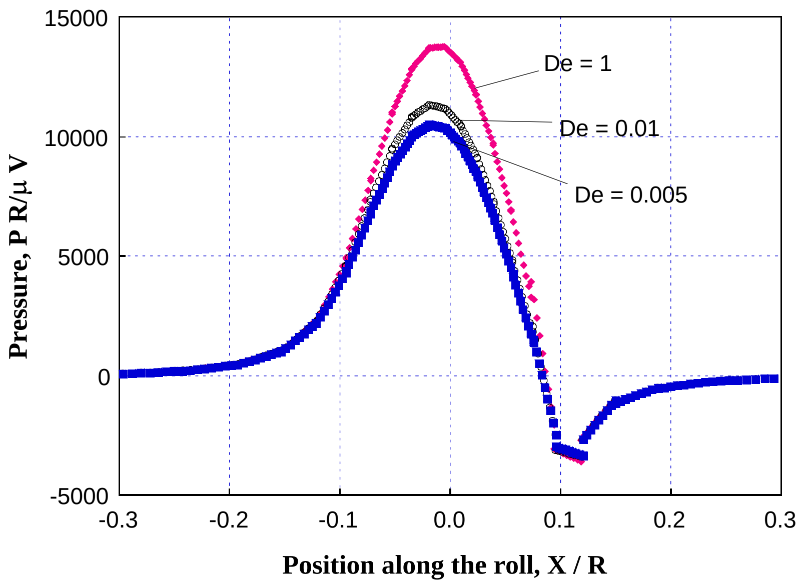

Figure 9: Pressure distribution at different Deborah numbers. H_0 / R = -0.002; Es = 10^{-4}; $\beta_1/(\beta_1+\beta_0)=0.3$.

The results show how the different parameters, including the viscoelastic properties of the rubber roll cover, can affect the performance of the roll nip. An important conclusion is that rolls with the same hardness, an elastic property obtained from a steady indentation test, can have completely different performance due to their viscoelastic behavior.

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